A SAT-Based Approach For PSPACE Modal Logics

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Abstract

SAT solvers have become efficient for solving NP-complete problems (and beyond). Usually those problems are solved by direct translation to SAT or by solving iteratively SAT problems in a procedure like CEGAR. Recently, a new recursive CEGAR loop working with two abstraction levels, called RECAR, was proposed and instantiated for modal logic K. We aim to complete this work for modal logics based on axioms (B), (D), (T), (4) and (5). Experimental results show that the approach is competitive against state-of-the-art solvers for modal logics K, KT and S4.

Introduction

SAT technology has proven to be a very successful practical approach to solve NP-Complete problems. One of the main issues is to find the "right" encoding for the problem, i.e. to find a polynomial reduction from the original problem into a propositional formula in Conjunctive Normal Form (CNF) which can be efficiently solved by a SAT solver. Recently, we proposed in (Lagniez et al. 2017), a recursive version of CEGAR in a procedure called RECAR (for Recursive Explore and Check Abstraction Refinement). We instantiated our framework for modal logic K. While the results obtained outperformed the other solvers in terms of instances solved, the solver was only designed to tackle modal logic K. Even if the other modal logics are PSPACE-Complete, in practice, the reductions are not straightforward and structural information can be lost during the translation process. In order to extend the scope of our framework to other modal logics, we propose to take advantage of the correspondence between modal logic axioms and structural constraints (Sahlqvist 1973) by encoding the common modal logic axioms (D), (T), (B), (4) and (5) into CNF. To this end, we complete the initial over-abstraction function by appending, for each axiom, a structural constraint which forces the Kripke structure to satisfy those axioms and we experimentally evaluate the proposed approach for modal logics KT and S4 for which we could find benchmarks.

Preliminaries

To introduce the required notions of modal logic, we define it, as usual, using Kripke semantics (Chellas 1980).

Definition 1 (Language of Modal Logic). *Let a countably infinite set of propositional variables $\mathbb{P}$ and a non-empty set of m unary modal operators $\mathbb{M} = \{\Box_1, \ldots, \Box_m\}$ be given. The language of modal logic (noted $\mathcal{L}$) is the set of formulas containing $\mathbb{P}$, closed under the set of propositional connectives $\{\neg, \wedge\}$ and the set of modal operators in $\mathbb{M}$. We also use the standard abbreviations $\Diamond_a \phi \stackrel{def}{=} \neg\Box_a\neg\phi$.*

Without loss of generality, we only consider modal logic formulas in negative normal form, noted NNF (negations appear only in front of propositional variables).

The depth of a formula ϕ in $\mathcal{L}$, denoted by depth(ϕ), is the highest number of nested modalities. The number of propositional variables in a formula ϕ in $\mathcal{L}$ is denoted by $Atom(\phi)$. Modal logic formulas can be satisfied by Kripke structures (Chellas 1980). The size of Kripke structures (the number of worlds) is bounded due to the Finite Model Property and the upper-bound is specific for each modal logic, as we can see below.

Lemma 1 (Sebastiani and McAllester 1997). *$UB(\phi) = Atom(\phi)^{\mathrm{depth}(\phi)}$ in modal logic K.*

Lemma 2 (Nguyen 1999). *$UB(\phi) = |\phi|^{2 \times |\phi| + \mathrm{depth}(\phi)}$ in modal logics KT and S4.*

RECAR for PSPACE Modal Logics

We redirect the reader to (Lagniez et al. 2017) to understand how works MoSaiC, but the idea behind is to translate a modal logic formula ϕ into a formula Σ in classical propositional logic (CPL) (Cook 1971) which addresses the question "is ϕ satisfied by a model of size n?". The n being bounded by a theoretical upper-bound $UB(\phi)$.

Encoding Modal Logic Axioms

It is well known that some axioms correspond to constraints on Kripke structures (Sahlqvist 1973). (T) is for reflexivity, (D) for seriality, (B) for symmetry, (4) for transitivity and (5) for euclideanity. Therefore, to deal with different modal logics, we simply append to the CPL's translation the following constraints corresponding to the different axioms (*m* corresponds to the number of modal operators and *n* to the number of worlds):

Solver #Instances	LWB_K SAT 504	LWB_K UNSAT 504	Total_K 1008	LWB_{KT} SAT 504	LWB_{KT} UNSAT 504	Total_{KT} 1008	LWB_{S4} SAT 504	LWB_{S4} UNSAT 504	Total_{S4} 1008
Moloss	71 (0)	83 (0)	154 (0)	68 (0)	170 (0)	238 (0)	269 (0)	203 (0)	472 (0)
InKreSAT	192 (24)	247 (0)	439 (24)	155 (9)	193 (0)	348 (9)	248 (0)	304 (0)	552 (0)
BDDTab	248 (5)	277 (4)	525 (9)	–	–	–	211 (0)	270 (0)	481 (0)
FaCT++	264 (10)	284 (19)	548 (29)	184 (30)	226 (59)	410 (89)	298 (42)	338 (25)	636 (67)
MoSaiC	263 (241)	306 (198)	569 (439)	**230** (251)	222 (253)	452 (504)	**277** (229)	225 (277)	502 (506)
$K_S P$	249 (4)	**328** (3)	577 (7)	130 (2)	93 (0)	223 (2)	223 (0)	205 (0)	428 (0)
Spartacus	**331** (33)	320 (10)	**651** (43)	207 (74)	**251** (59)	**458** (133)	273 (17)	**350** (13)	**623** (30)
VBS	340	328	668	230	251	481	277	352	629

Table 1: Number of LWB instances solved in K, KT and S4

Definition 2 (Translation of Axioms).

$$\text{over}((T), n) = \bigwedge_{a=0}^{m} \bigwedge_{i=0}^{n} (r_{i,i}^{a})$$

$$\text{over}((D), n) = \bigwedge_{a=0}^{m} \bigwedge_{i=0}^{n} \bigvee_{j=0}^{n} (r_{i,j}^{a})$$

$$\text{over}((B), n) = \bigwedge_{a=0}^{m} \bigwedge_{i=0}^{n} \bigwedge_{j=0}^{n} (r_{i,j}^{a} \to r_{j,i}^{a})$$

$$\text{over}((4), n) = \bigwedge_{a=0}^{m} \bigwedge_{i=0}^{n} \bigwedge_{j=0}^{n} \bigwedge_{k=0}^{n} ((r_{i,j}^{a} \wedge r_{j,k}^{a}) \to r_{i,k}^{a})$$

$$\text{over}((5), n) = \bigwedge_{a=0}^{m} \bigwedge_{i=0}^{n} \bigwedge_{j=0}^{n} \bigwedge_{k=0}^{n} ((r_{i,j}^{a} \wedge r_{i,k}^{a}) \to r_{j,k}^{a})$$

Each $r_{i,j}^{a}$ meaning that the w_j is accessible from w_i by the relation a. Thus, when axiom (T) is considered (i.e. modal logic KT), the over-abstraction function is (over$(\phi, n) \wedge$ over$((T), n)$). When both axioms (T) and (4) (i.e. modal logic S4) are considered, the over-abstraction function is (over$(\phi, n) \wedge$ over$((T), n) \wedge$ over$((4), n)$).

Experimental results

We chose to compare the solvers on the classical LWB benchmarks for modal logics K, KT and S4 (Balsiger, Heuerding, and Schwendimann 2000). Indeed, we already saw that MoSaiC was the fastest solver in (Lagniez et al. 2017) on many different benchmarks in K, so we wanted to close the gap of our weakness: LWB. The experiments ran on a cluster of Xeon, 4 cores, 3.3 GHz with CentOS 6.4 with a memory limit of 32GB and a runtime limit of 900 seconds per solver per benchmark, no matter the logic considered.

We compared MoSaiC against state-of-the-art solvers for the modal logics K, KT and S4, namely: Moloss 0.9, $K_S P$ 0.1.2, BDDTab 1.0, FaCT++ 1.6.4, InKreSAT 1.0, Spartacus 1.1.3. It is important to notice that $K_S P$ (kindly provided by its authors) is still under development for modal logics KT and S4. Its results should be considered as preliminary.

We can see on Table 1 that MoSaiC is competitive. Indeed it is able to solve more satisfiable benchmarks in KT and S4 and still being competitive on unsatisfiable ones. It is worth remembering that the approach here is generic compared to other solvers dedicated only to K, KT and S4. The main weakness of MoSaiC is its memory consumption, it does not solve problems mainly due to the fact that it goes above the 32GB limit.

Conclusion

In this article, we presented how MoSaiC can be extended to deal with other modal logics than K. We show that just translating the axioms leads to a competitive solver (especially in KT) but not a better one. In the future, we will work directly on a new RECAR instantiation which will be axiom-dependent to solve fastly instances, the drawback of such a technique is being less generic than the approach presented here.

Acknowledgement

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