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► **To cite this version:**

Hamad Hazim, B. Rousselet. Frequency sweep for a beam system with local unilateral contact modeling satellite solar arrays. Tendances des applications mathématiques en Tunisie, Algerie, Maroc, May 2009, Morocco. pp.541. hal-00418507v1

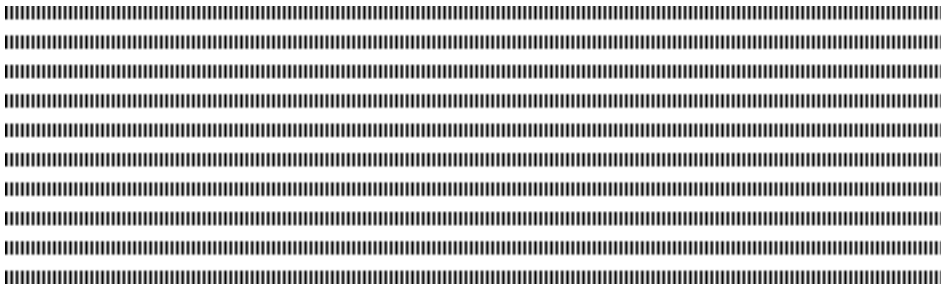
**HAL Id: hal-00418507**

**<https://hal.univ-cotedazur.fr/hal-00418507v1>**

Submitted on 30 Sep 2009 (v1), last revised 11 Oct 2009 (v2)

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TAM-TAM, Maroc, Kenitra 6-8 may 2009

## Frequency sweep for a beam system with local unilateral contact modeling satellite solar arrays

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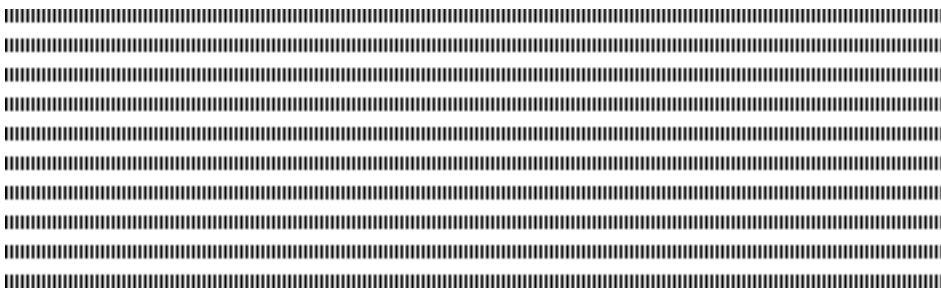


**RÉSUMÉ.** Dans le but de réduire la masse des panneaux solaires d'un satellite, la flexibilité de ces panneaux est devenue non négligeable et ils peuvent se taper l'un contre l'autre ce qui peut provoquer un endommagement de la structure. Pour prévenir cela, des cales sont fixées dans des positions bien choisies de la structure; ils jouent le rôle d'un ressort élastique unilatéral. Comme conséquence négative, la dynamique de ces panneaux devient nonlinéaire. Une approximation par éléments finis est utilisée pour résoudre les équations aux dérivées partielles qui gouvernent la dynamique de la structure; des balayages en fréquence ont été faits numériquement pour étudier le comportement dynamique. Les modes normaux non linéaires sont à l'étude.

**ABSTRACT.** In order to save mass of satellite solar arrays, the flexibility of the panels becomes not negligible and they may strike each other; this may damage the structure. To prevent this, rubber snubbers are mounted at well chosen points of the structure and they act as **one sided** linear spring; as a negative consequence, the dynamic of these panels becomes **nonlinear**. The finite element approximation is used to solve partial differential equations governing the structural dynamic. Frequency sweep has been performed numerically to study the dynamic behavior. Non linear normal modes are under study

**MOTS-CLÉS :** Dynamique non linéaire, Systèmes différentiels, Modélisation

**KEYWORDS :** Nonlinear dynamics, Differential systems, Modeling



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## 1. Introduction

In articles [1], [3], [4] and [9], the dynamic of a beam system with a nonlinear contact force, under a periodic excitation given as an imposed acceleration form is studied both numerically and experimentally. When sweeping frequencies in an interval which contains eigen frequencies of the beam, resonance phenomena appear as well as new frequencies caused by the unilateral contact.

The study of the total dynamic behavior of solar arrays in a folded position with snubbers are so complicated, that to simplify, a solar array is modeled by a clamped-free Bernoulli beam with one-sided linear spring. This system is fixed on a shaker which has a vibratory motion  $d(t)$ , it is given as a periodic imposed acceleration :  $\ddot{d}(t) = a \sin \omega t$  (see figure 1).

The present study is to simulate the behavior of a beam which strikes a snubber. In this first step, existing algorithms are used in order to prepare a sequence of experiments. The focus is on the comparison between the linear and the non-linear case.

An original approach of an extension of normal modes to this non-linear case is under study ; this approach is an alternative to the one of [2] and [3]

The beam motion with a snubber can be modeled as :

$$\rho S \ddot{u}(x, t) + EI u^{(iv)}(x, t) = 0 \quad [1]$$

with the boundary conditions :

$$u(0, t) = d(t), \quad \partial_x u(0, t) = 0, \\ EI u^{(2)}(L, t) = 0 \text{ and } EI u^{(3)}(L, t) = k_r (d(t) - u(L, t))_+.$$

$$u(x, t)_+ = \begin{cases} u(x, t) & \text{if } u > 0 \\ 0 & \text{if } u \leq 0 \end{cases} \quad [2]$$

The classical Hermite cubic finite element approximation is used, it yields an ordinary differential system in the form :

$$M \ddot{q} + K q = k_r (d(t) - q_n)_+ e_n \quad [3]$$

Where  $M$  and  $K$  are respectively the mass and stiffness assembled matrices,  $q$  is the vector of degrees of freedom of the beam,  $q_i = (u_i, \partial_x u_i)$ ,  $i = 1, \dots, n$ , where  $n$  is the size of  $M$ .

The system is integrated numerically using the Scilab routine 'ODE' for stiff problems, package ODEPACK is called and it uses the BDF method [8].

The frequency sweep is performed by running the integration of ordinary differential systems at a chosen initial value of the frequency and saving the maximum of the displacement of all nodes in the whole time interval. The frequency is then incremented till the

end of the frequency interval and the same procedure is used again.

The maximum of displacement is plotted in the frequency domain, the simulation shows many resonance phenomena when approaching particular frequencies ; these particular frequencies are the non-linear frequencies of the system corresponding to the eigen frequencies of the linearized system (bilateral spring). Other superharmonic and subharmonic frequencies appear due to the nonlinearity of the contact. The frequency responses are compared with the FFT of the temporal signal coming from the direct integration of the differential systems. On the other hand, the calculation of the **non-linear normal modes (NNM)** [3] and [2] will be a theoretical way to validate the numerical simulations.

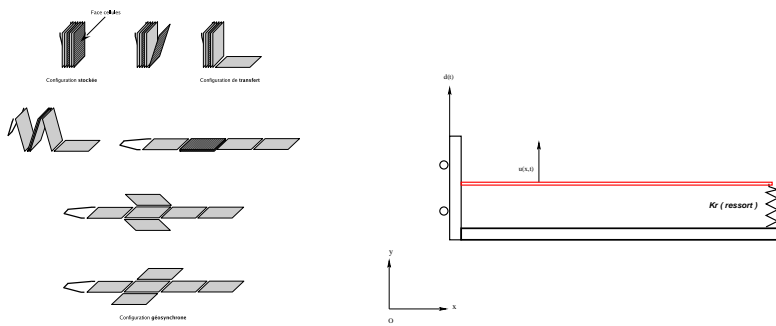


Figure 1 – At left : solar arrays from folded to final position, at right : beam system with unilateral spring on a shaker to model an unfolded array

## 2. Numerical results

The Physical properties and the dimensions of the beam with a spring stiffness of  $5 \times 10^5 N/m$  gives the following eigen-frequencies (bilateral spring) :

- *first frequency* =  $196.35694 Hz$       *second frequency* =  $472.07584 Hz$
- *third frequency* =  $961.52462 Hz$

The figure (2) show the eigen-frequencies of the linearised system (bilateral spring) calculated analytically and the corresponding frequencies of the non-linear cases (unilateral spring); many other frequencies appear in the non-linear cases due to the nonlinearity of the contact. The superharmonics and the subharmonics frequencies are shown too, the superharmonics are the double and the triple (etc..) of the corresponding eigen frequency, the subharmonics are the half and the third (etc..) of the eigen frequencies. Many other combinations of these frequencies may exist but it is quit difficult to distinguish their location.

A damping term in the spring will help to distinguish each mode with a particular initial

condition or excitation, the full article will show this technique.

The numerical results are to be compared with experiments in preparation.

On the other hand, the calculation of the non-linear normal modes are under development in order to address more precisely the frequency response of the structure and to validate numerical results.

Comparison with results obtained by asymptotic expansions of the type in [6] is in project

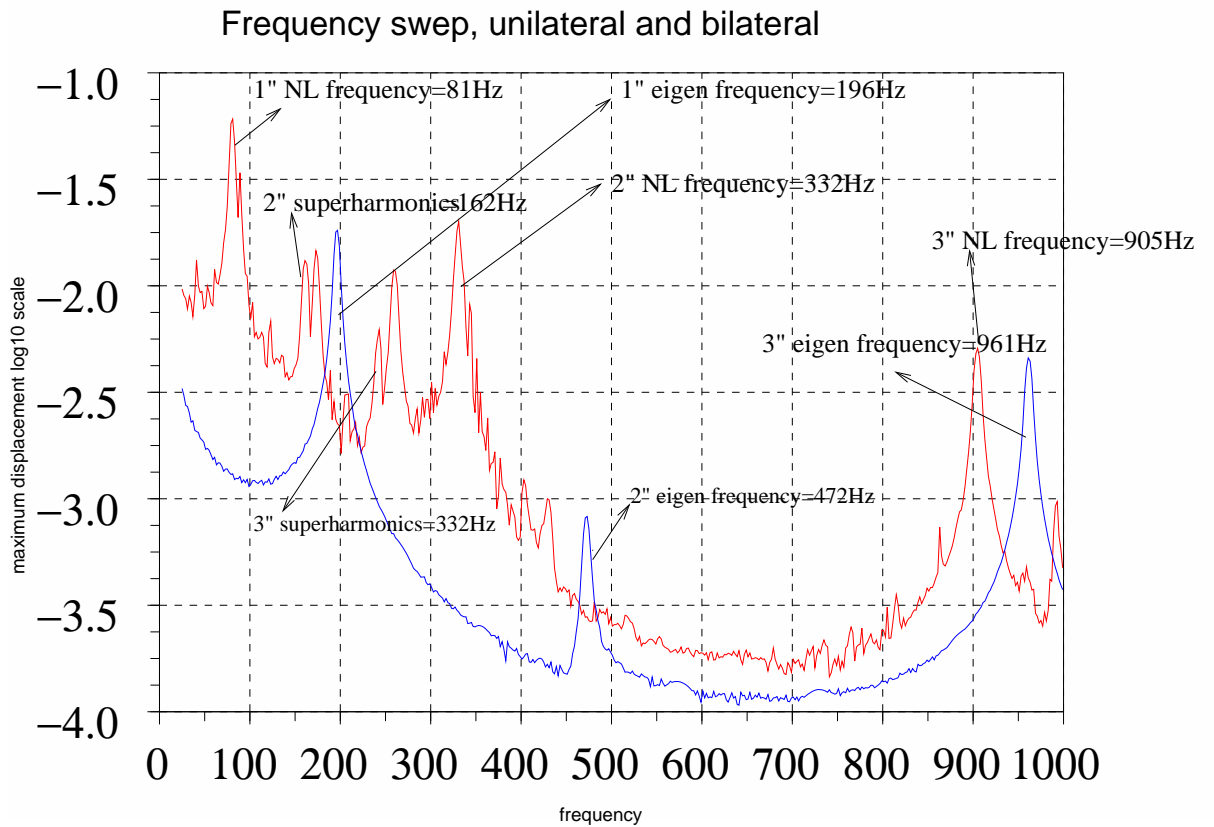


Figure 2 – Frequency sweep, bilateral and unilateral spring, no pres stress or backlash,  $beam\ height = 10mm, kr = 500.000N/m, a = 50m/s^2, tf = 0.4s$

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